

# A probabilistic description scheme for rotating machinery health evaluation<sup>†</sup>

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## Abstract

Condition-based maintenance has become more popular in recent years because of its advantages in terms of minimizing downtime, extending lifetime, and reducing cost. This kind of maintenance strategy is based on condition monitoring of machinery in operation. Condition monitoring is a key step in maintenance decision analysis. Numerous non-stationary signal processing methods have been developed to reveal fault characteristics in rotating machinery. In this study, an adaptive signal analysis method called empirical mode decomposition is employed for gearbox vibration signal preprocessing. Considering a modulation phenomenon that appeared in a faulty gear, the Hilbert Transform is applied to obtain an envelope signature, which usually contains abundant fault-related signatures. Being different from other failure classification problems, this paper is concerned with determining the probability of normal condition based on current observations describing the condition of a gearbox. Moreover, according to Bayes rule, this problem can be translated to estimate the conditional probability of current observations given normal gearbox condition using a Hidden Markov Model. From this point, a novel probabilistic health description index called Average Probability Index is proposed for gearbox health evaluation. For automatic detection, a semi-dynamic threshold is presented to detect an early fault in a gear. At last, validation and comparative studies are performed using two sets of gearbox lifetime accelerated testing vibration data. The results show the advantages of the proposed method for gearbox condition monitoring.

*Keywords:* Condition-based maintenance; Health evaluation; Empirical mode decomposition; Average probability index; Hidden Markov model

## 1. Introduction

Machinery suffers from many kinds of failures during operation. Thus, research on maintenance becomes necessary to guarantee the reliability of machinery. The history of maintenance engineering can be traced back to run-to-failure maintenance, which does not implement maintenance actions until machinery breakdown. However, this kind of maintenance strategy has many disadvantages, including high downtime, equipment damage, high repair cost, etc. Later, time-based preventive maintenance, which is also called planned maintenance, was developed to perform maintenance activities on the basis of machinery failure statistics. Since it does not consider the current health condition of machinery, time-based preventive maintenance cannot avoid random breakdowns and can even make the total maintenance cost higher. Therefore, it is crucial to explore a new strategy to overcome the aforemen-

tioned disadvantages. Fortunately, a very useful strategy called Condition-based Maintenance (CBM) has been developed to implement a planned maintenance schedule. CBM regards the health condition of machinery as the basis for maintenance decision-making. Therefore, the proper method to monitor the health condition of machinery is a vital question in CBM and is the motivation for this study [1].

Fig. 1 illustrates three main steps in CBM, which can also be treated as three modules: data acquisition, data processing, and maintenance decision-making. First, original signals, such as vibration information, are obtained by using sensors. Second, these signals are processed to extract or reveal fault signatures in order to check the health condition of machinery. Third, effective maintenance policies are planned [1]. Data processing is the key step that extracts critical information from abundant redundancy data collected by the data acquisition module, and the output of it determines the performance of the maintenance decision-making module.

Many data processing methods have been developed to reveal fault characteristics of non-stationary mechanical vibration signals. Among all these available methods, the Wavelet

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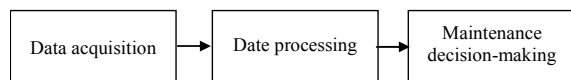


Fig. 1. A general flow chart of CBM program.

Transform (WT) [2] is widely used in mechanical fault detection. The main advantage of WT is its excellent performance with good time resolution at high frequency and good frequency resolution at low frequency. However, the application of WT has limitations. It is hard to determine which wavelet should be used to measure the similarity between the original signal and the wavelet function and to determine how many decomposition levels are needed. Further, once both wavelet function and decomposition levels are determined, they will be applied to the whole analysis process. That is to say, this method is a non-adaptive signal processing technique. There is also an energy leakage problem owing to the limited length of the wavelet function.

In recent years, another novel adaptive time series analysis technique called Empirical Mode Decomposition (EMD) [3] has been developed to analyze non-stationary and non-linear signals. This method is capable of decomposing a signal into a number of Intrinsic Mode Functions (IMFs) according to the local characteristic time/scale of the signal itself. Therefore, the decomposition process is a self-adaptive procedure. What is more, the generated IMFs are stationary. Due to this potential in signal processing, a lot of research has been conducted recently [4-9]. For example, Fan and Zuo [4] employed EMD to decompose raw vibration signals into IMFs, which is a self-adaptive program to detect machine faults at the earliest onset of deterioration. Chen et al. [5] presented an ensemble empirical mode decomposition method in pipeline corrosion inspection. Dong et al. [6] improved the sifting process of EMD for bearing fault diagnosis. Bassiuny et al. [7] employed EMD for fault diagnosis of the stamping process. Gao et al. [8] used EMD to decompose a signal into IMFs and presented the concept of Combined Mode Function (CMF) to solve the failure of IMFs to reveal signal characteristics in case of noise. Li et al. [9] proposed a new Ranged Angle-Empirical Mode Decomposition method to identify the vital components of a diesel engine that have abnormal clearance.

Furthermore, the phenomena of signal modulation exist due to the occurrence of local faults in the process of condition monitoring. Since EMD decomposes multi-component amplitude and frequency-modulated signals, and since each obtained IMF may be amplitude-modulated or frequency-modulated, it is necessary to apply the Hilbert Transform (HT) for signal demodulation.

With the motivation of emphasizing the capability of EMD in early fault detection, we conducted research to determine EMD's behavior for online health evaluation of machinery. This is different from the work of others [4-9], as they focused on applying EMD to distinguish different machinery fault patterns, while the goal of our study is to realize online health description with a probabilistic scheme based on EMD and the

Hidden Markov Model (HMM).

In this study we extract fault features using a combination of EMD, CMF, and HT. Next, a novel condition monitoring curve, which is described by the Average Probability Index (API), is constructed using HMM due to its success in speech recognition [10]. Moreover, this application of HMM has great novelty because traditional applications of HMM in CBM mostly concentrate on fault diagnosis rather than prognosis [11-13]. Then, a kind of semi-dynamic threshold is defined to detect the occurrence of an early fault. That is to say, an early fault happens when the proposed index API is lower than its corresponding dynamic threshold.

The rest of this paper is organized as follows. Section 2 describes a signal processing method using Empirical Mode Decomposition. In Section 3, a novel health evaluation index for a gearbox and its semi-dynamic threshold is proposed. In Section 4, two sets of gearbox vibration data provided by the Applied Research Laboratory at Pennsylvania State University are used to validate the proposed method and comparisons are performed. Conclusions are presented in the end.

## 2. Signal decomposition scheme

### 2.1 Empirical mode decomposition

The basic idea of Empirical Mode Decomposition is to decompose original signals into different Intrinsic Mode Function (IMF) components. Each IMF must satisfy the following conditions [3]: (1) in the whole data set, the number of extrema and zero-crossings must either equal or differ at most by one. (2) At any point, the mean value of the envelopes defined by local maxima and local minima is zero.

An IMF expresses a simple oscillatory mode compared with a simple harmonic function. The decomposition process of any signal  $x(t)$  can be described as follows:

(1) Connect all the local maxima of signals using a cubic spline line. The connected line can be called the upper envelope,  $u_1$ .

(2) Repeat the process for local minima to obtain the lower envelope,  $l_1$ . The upper and lower envelopes should cover all of the data between them.

(3) The mean of the upper and lower envelopes' values is  $m_1$ , so the first residual  $h_1$  can be obtained as follows:

$$m_1 = \frac{u_1 + l_1}{2} \quad (1)$$

$$h_1 = x(t) - m_1 \quad (2)$$

(4) Ideally, if  $h_1$  is an IMF, then it is the first component of  $x(t)$ . However, if  $h_1$  is not an IMF, it is regarded as the original signal, and steps (1)- (3) are repeated. That is,

$$h_{11} = h_1 - m_{11} \quad (3)$$

Repeat sifting, i.e. up to  $k$  times until  $h_{1k}$  becomes an IMF, that is,

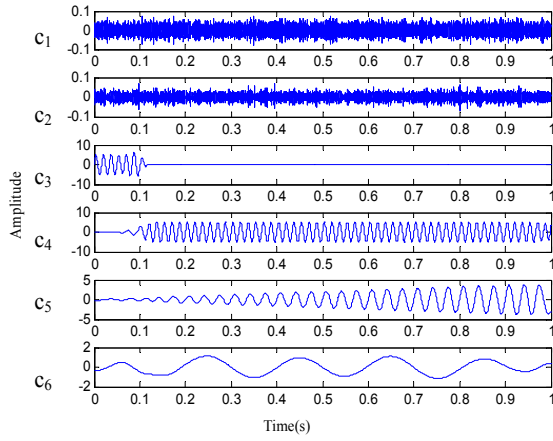


Fig. 2. EMD decomposition results of  $x_1(t)$ .

$$h_{1k} = h_{1(k-1)} - m_{1k} . \tag{4}$$

Then the obtained IMF can be designated as:

$$c_1 = h_{1k} . \tag{5}$$

(5) Separate  $c_1$  from  $x(t)$  and the residual  $r_1$  can be given as

$$r_1 = x(t) - c_1 . \tag{6}$$

(6) Treat  $r_1$  as the original data, and repeat the process as described above  $n$  times. As a result,  $n$ -IMF of signal  $x(t)$  can be obtained as follows:

$$\begin{aligned} r_1 - c_2 &= r_2 \\ \vdots \\ r_{n-1} - c_n &= r_n . \end{aligned} \tag{7}$$

The decomposition process does not stop until the residual  $r_n$  becomes a monotonic function or a constant from which no more IMF components can be extracted. To sum up (6) and (7), we can get:

$$x(t) = \sum_{i=1}^n c_i(t) + r_n(t) . \tag{8}$$

### 2.2 Construction of combined mode function

In machinery condition monitoring, a critical problem is the impact of noise, which may be caused by certain systematic factors such as sensor drifting, measurement error, etc. The existence of noise leads to distortion in EMD decomposition. In order to demonstrate this phenomenon, a simulated signal  $x_1(t)$  is defined by adding a noise component. That is,

$$\begin{aligned} x_1(t) &= 5 \sin(120\pi t) + 7 \sin(60\pi t) \sin(0.2\pi t) \\ &\quad + \sin(10\pi t) + 0.035 \text{rand}(t) \quad t \in [0,1] \end{aligned} \tag{9}$$

where  $\text{rand}(t)$  is normally distributed white noise. The analy-

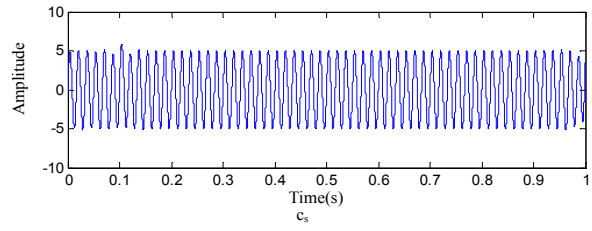


Fig. 3. A CMF  $c_s$  by combining  $c_3$  and  $c_4$ .

sis results of  $x_1(t)$  using EMD are shown in Fig. 2. It is obvious that  $c_4, c_5$  and  $c_6$  are the major components of  $x_1(t)$ . However, the existence of noise affects the decomposition results and  $c_4$  is distorted locally.

In order to deal with this problem, the concept of Combined Mode Function (CMF) [8] is applied in this paper. The idea of CMF is to mix neighboring IMFs to obtain a CMF, and this operation increases the accuracy of EMD. Generally, CMF  $c_s$  is given by:

$$c_s = c_i + c_{i+1} + \dots + c_{i+m} \tag{10}$$

where  $1 \leq i \leq n-m, n$  is the maximal number of IMFs obtained using EMD. In this example, a new CMF, as shown in Fig. 3, is realized by combining  $c_3$  and  $c_4$ . Through this operation, the distortion caused by noise can be eliminated.

### 2.3 Demodulation with Hilbert Transform

In rotating machinery vibration analysis, another important issue is the impact of signal modulation. For example, in case a tooth failure occurs on a rotating gear, meshing frequency (caused by gear teeth meshing) is modulated by rotating frequency (related to the existence of a fault), which results in the emergence of sidebands around the meshing frequency and its harmonics in spectrum analysis. According to the analysis results in Sections 2.1 and 2.2, each obtained IMF may be amplitude-modulated or frequency-modulated after a multi-component amplitude-modulated or frequency-modulated signal is decomposed using EMD. Therefore, in order to overcome modulation caused by faults, the Hilbert Transform is employed to reveal fault signatures.

For time series  $x(t)$ , its Hilbert Transform,  $H[x(t)]$ , which is a time-domain convolution that maps one real-valued time-history into another, is defined as:

$$H[x(t)] = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t - \tau} d\tau . \tag{11}$$

Here,  $t$  and  $\tau$  are the time and translation parameters, respectively. It is known that the Hilbert Transform is a frequency-independent  $90^\circ$  phase shifter. Thus, this method does not affect the non-stationary characteristics of modulated signals, which may be caused by machine faults. Demodulation can be accomplished by forming a complex-valued time-domain signal called an analytic signal, which is given by

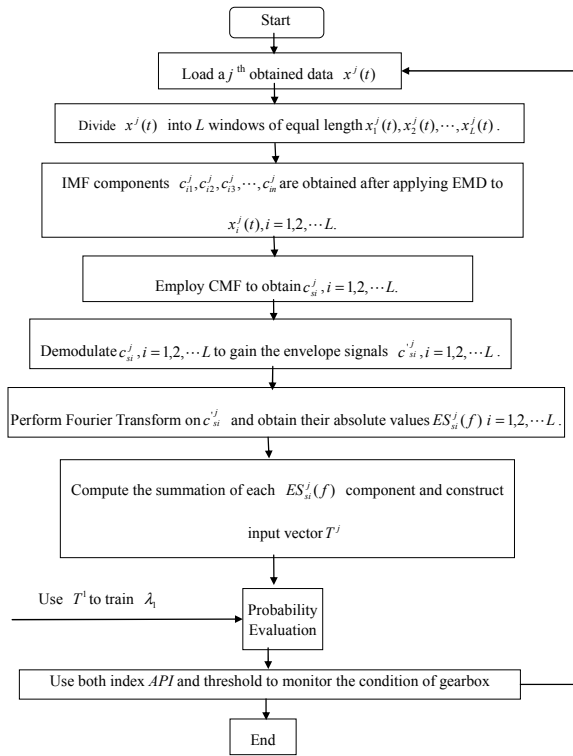


Fig. 4. The flowchart of the average probability index.

$$B(t) = x(t) + iH[x(t)] = b(t)e^{i\phi(t)} \tag{12}$$

where  $b(t) = \sqrt{x^2(t) + H^2[x(t)]}$ ,  $\phi(t) = \arctan \frac{H[x(t)]}{x(t)}$ , and  $i = \sqrt{-1}$ ;  $b(t)$  is the envelope of  $B(t)$ .

### 3. Condition evaluation with health index

In order to provide useful information for CBM maintenance decisions, it is necessary to establish a condition monitoring system that can give an actual description of machinery health conditions. Our work described in the previous section mainly focused on signal processing methods, including decomposition of vibration signal with EMD and selection of the appropriate CMF for further analysis. In this section, a novel index called the Average Probability Index is defined on the basis of the selected CMF, which gives a quantitative description of machinery health conditions. The proposed API is constructed using HMM. The key advantage of this method is its adaptiveness, owing to the application of an adaptive signal processing method, EMD, for signal decomposition. Furthermore, to identify the occurrence of an early fault, a semi-dynamic threshold criterion corresponding to this API index is proposed. In order to better understand the work conducted in this paper, Fig. 4 shows the flowchart of the proposed method.

#### 3.1 Extraction of characteristic features

In the implementation of Condition-based Maintenance, on-

line condition monitoring is usually expected, since it can provide timely information for maintenance decision analysis. In this research, online condition monitoring is realized by collecting vibration data at a certain time interval (e.g., once every 30 minutes) and numbered consequently. Therefore,  $x^j(t)$  represents a piece of signal collected at a certain time point  $j$ .

Assume that the signal  $x^j(t)$  collected at time point  $j$  is divided into  $L$  windows of equal length  $x_1^j(t), x_2^j(t), \dots, x_L^j(t)$ . According to the decomposition procedure described in Section 2,  $L$  pieces of demodulated CMFs can be obtained and are termed  $c_{si}^j, i = 1, 2, \dots, L$ . The first five steps in Fig. 4 describe this decomposition process, which includes three main tasks: EMD decomposition, construction of CMF, and demodulation with a Hilbert Transform.

To extract fault features from demodulated  $c_{si}^j$ , spectrum analysis is performed with a Fourier Transform as follows:

$$ES_{si}^j(f) = \left| \int_{-\infty}^{+\infty} c_{si}^j(t) e^{-2\pi i f t} dt \right| \quad i = 1, 2, \dots, L. \tag{13}$$

The result of the Fourier Transform reflects the energy distribution of a signal in the frequency domain. In the case of gearbox condition monitoring, the existence of a fault results in a change of energy. Therefore, in the context of gear fault detection, the summation of these amplitude values of the envelope spectrum will increase greatly for a faulty gear compared with a normal gear. Thus, a new term is defined to describe the total energy of  $c_{si}^j$ ,

$$SES_{si}^j = \log_{10}(\text{sum}(ES_{si}^j(f))) \quad i = 1, 2, \dots, L. \tag{14}$$

Finally, a new feature  $T^j$  can be extracted from the vibration data collected at time  $j$  as follows, which includes  $L$  feature vectors.

$$\begin{aligned} T_1^j &= [SES_{s1}^j, SES_{s2}^j, \dots, SES_{sL}^j] \\ T_2^j &= [SES_{s2}^j, SES_{s3}^j, \dots, SES_{s1}^j] \\ T_3^j &= [SES_{s3}^j, SES_{s4}^j, \dots, SES_{s2}^j] \\ &\vdots \end{aligned} \tag{15}$$

$$\begin{aligned} T_L^j &= [SES_{sL}^j, SES_{s1}^j, \dots, SES_{s(L-1)}^j] \\ T^j &= [T_1^j, T_2^j, \dots, T_L^j]. \end{aligned} \tag{16}$$

#### 3.2 Definition of API based on HMM

When features have been extracted, the proposed health index can be defined using Hidden Markov Model. The HMM is a doubly embedded stochastic model that has been widely applied in pattern recognition, including fault classification. However, the work with HMM in this paper is different from others, since we utilize HMM for the definition of health index rather than fault classification. Generally, an HMM can be represented by a compact notation  $\lambda = \{A, B, \pi\}$ . For more details about HMM, the readers is referred to reference [10].



Fig. 5. Mechanical diagnostic test bed.

To define API, assume that the hidden states are governed by a homogeneous Markov chain of order 1. Here, we use  $T^1$  to train HMM  $\lambda_1$ , which represents the normal condition of the gearbox. The purpose of HMM training is to estimate the model parameters  $\lambda = \{A, B, \pi\}$  and the training process is realized by fitting the observation probability distribution with Gaussian mixture models (GMMs) for each of the Q states using K-means. After that, improve the parameter estimates of HMM with a mixture of Gaussian outputs using the Baum-Welch re-estimation algorithm [10]. Once the HMM for normal conditions is trained, the remaining features,  $T^j$  s, are used as validation data. Since each feature extracted at time  $j$  includes  $L$  feature vectors,  $T^j = [T_1^j, T_2^j, \dots, T_L^j]$ , we can get a vector of output probability termed as  $G^j = [G_1^j, G_2^j, \dots, G_L^j]$ . Therefore, a novel index called the Average Probability Index (API) is proposed as follows:

$$API(j) = \frac{G_1^j + G_2^j + \dots + G_L^j}{L} \quad j = 1, 2, \dots, n \quad (17)$$

Here, variable  $j$  corresponds to the time point during vibration data collection. Since the proposed health index is a statistic of probability, we call it a probabilistic scheme for machinery health evaluation. Furthermore, since the lifetime gearbox vibration data used in the case study are collected and numbered sequentially at a certain time interval (e.g., once half an hour) along with the running of the test, time  $j$  is consistent with the file number and is represented by the file number  $(1, 2, \dots, n)$ .

In order to detect the occurrence of a gear fault, a semi-dynamic threshold  $Th(t)$  is defined as below, based on the principle of a 3-sigma limit of time series data:

$$Th(t) = \begin{cases} \frac{\sum_{b=1}^t API(b)}{t} - 3 \times \left( \frac{\sum_{b=1}^t (API(b) - \frac{\sum_{b=1}^t API(b)}{t})^2}{t-1} \right)^{1/2} & t = 1, 2, \dots, t_{early}, \\ \frac{\sum_{b=1}^{t_{early}} API(b)}{t_{early}} - 3 \times \left( \frac{\sum_{b=1}^{t_{early}} (API(b) - \frac{\sum_{b=1}^{t_{early}} API(b)}{t_{early}})^2}{t_{early}-1} \right)^{1/2} & t = t_{early} + 1, t_{early} + 2, \dots, n. \end{cases} \quad (18)$$

Here, semi-dynamic means that the proposed threshold

Table 1. Time information of workload change in TR#5.

	Time period	Time stamp	File number	Workload
Condition #1	6/19/97 13:35 (GMT) – 6/23/97 13:35 (GMT)	000-176	1-12	540 in-lbs (100%max)
Condition #2	6/23/97 13:35 (GMT) – 6/24/97 20:56 (GMT)	192-262	13-83	1620 in-lbs (300%max)

Table 2. Time information of workload change in TR#10.

	Time period	Time stamp	File number	Workload
Condition #1	11/17/97 16:20 (GMT) – 11/21/97 16:20 (GMT)	000-190	1-12	540 in-lbs (100%max)
Condition #2	11/21/97 16:20 (GMT) – 11/25/97 13:25 (GMT)	195-387	13-148	1080 in-lbs (200%max)

Table 3. Gearbox information of TR#5 and TR#10.

Gearbox ID#	DS3S0150XX
Make	Dodge APG
Model	R86001
Gear Ratio	1.533
Contact Ratio	2.388
Number of Teeth (Driven gear)	46
Number of Teeth (Driving pinion)	30
Meshing Frequency	875.53 (Hz)

changes with the historical value of API before the occurrence of an early fault. If the current API is lower than the corresponding threshold at the first time ( $t_{early}$ ), we conclude that an incipient fault has occurred. Then, the semi-dynamic threshold becomes a fixed value.

## 4. Experimental validation

### 4.1 Experimental setup

In this study, two sets of vibration data collected from a mechanical diagnostic test bed (MDTB) (shown in Fig. 5) were used to validate the proposed method. These vibration data were provided by the Applied Research Laboratory at Pennsylvania State University from two test runs of single reduction helical gearboxes, which are named as TR#5 and TR#10, respectively.

The experiment started with a brand new gearbox under 100% of the rated workload (Condition #1). After some time, the workload was doubled or tripled (Condition #2), and the test rig was shut down as a result of two accelerometers exceeding a predetermined limit of 150% of the root mean square. There were a series of inspections performed in the process of each test run. Each inspection generates a piece of signal collected in a 10-second window at a sampling rate of 20 kHz. The signal is saved as a data file and numbered se-

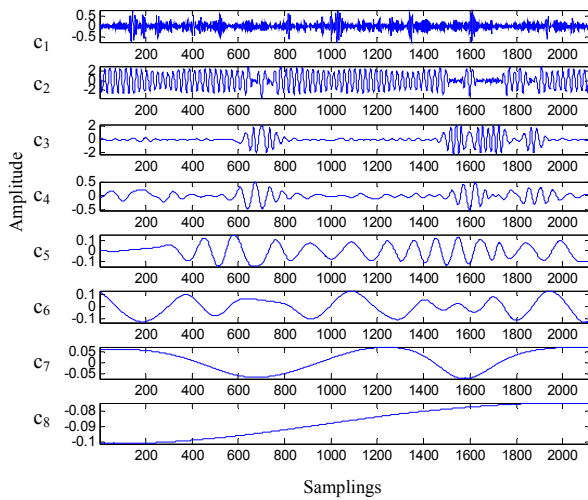


Fig. 6. The results of normal gear condition with two revolutions using EMD from TR#5.

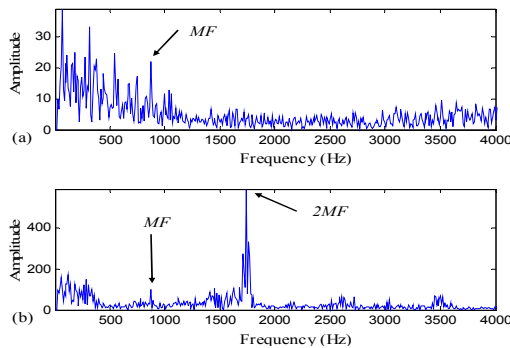


Fig. 7. Frequency domain exhibition of  $c_1$  and  $c_2$  with two revolutions. (a) Frequency spectrum of signal  $c_1$ ; (b) frequency spectrum of signal  $c_2$ .

quentially. Tables 1-2 provide the time specifications of TR#5 and TR#10. The mechanical specifications of the gearbox are given in Table 3.

#### 4.2 Signal adaptive decomposition and construction of CMF

First of all, API should be emphasized for its adaptive quality because EMD itself possesses an adaptive characteristic in the process of decomposition, compared with Wavelet Transform, Wavelet Packet Transform, and Wavelet Lifting Scheme, which means that there is no need to set the parameters, such as decomposition level, basis function, and so on, to decompose the original signal. To validate the proposed method, we first investigated the performance of the signal decomposition method proposed in this research. Fig. 6 shows the decomposition results of normal gear conditions with two revolutions from TR#5. Through decomposition, 8 IMFs can be obtained, and it is obvious that both  $c_1$  and  $c_2$  include high frequency components that correspond to the meshing frequency and its harmonics.

Fig. 7 demonstrates that the first two components include the meshing frequency and its harmonics signatures. However, we

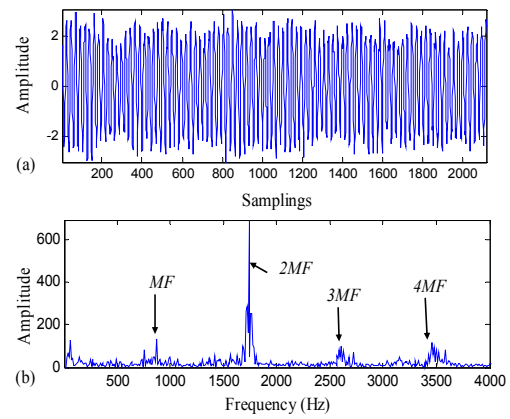


Fig. 8. The signal of CMF and its spectrum with two revolutions. (a) The signal by combining the first four IMFs; (b) its frequency spectrum.

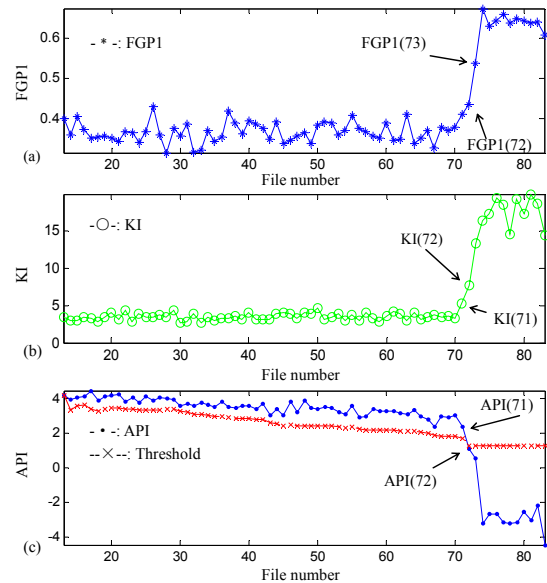


Fig. 9. Comparison of KI, API, and FGPI using TR#5 under constant torque. (a) Health evaluation using FGPI; (b) health evaluation using KI; (c) health evaluation using API.

can also observe that  $c_2$  suffers from local distortions between both 600–800 samplings and 1500–1900 samplings. These distortions usually result from noise, and the reasons have been given in the Section 2. On the other hand, we also observe that both  $c_3$  and  $c_4$  seem to be the losing part of  $c_2$ . As a result, in order to mitigate the influence of noise on meshing frequency and its harmonics in later analysis, we combined the first four IMFs into one CMF. Both the time domain signal of CMF and its corresponding frequency spectrum are shown in Fig. 8.

#### 4.3 Evaluation of the proposed Average Probability Index API

##### 4.3.1 Construction of API with constant number of states and GMMs.

To begin, we chose a normal data file from TR#5 to represent the gear's normal status. According to the feature extrac-

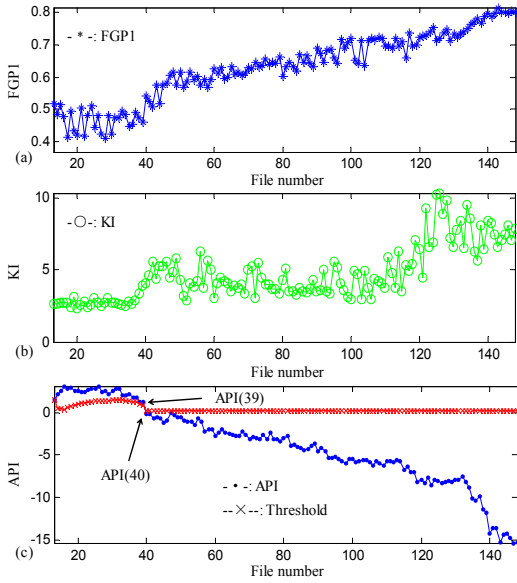


Fig. 10. Comparison of KI, API, and FGP1 using TR#10 under constant torque. (a) Health evaluation using FGP1; (b) health evaluation using KI; (c) health evaluation using API.

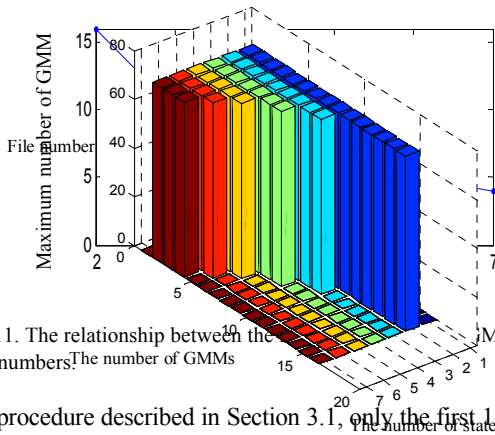


Fig. 11. The relationship between the maximum number of GMMs and state numbers.

tion procedure described in Section 3.1, only the first 16 revolution samplings in each piece of the signal were needed to reduce the online computing time. Then, we divided 16 revolution samplings into 8 equal length signals, with  $L=8$ . Each of these contained 218 sampling points, which included 2 revolutions. The decomposition process was introduced in Section 4.2. After we obtained the CMF, demodulation was needed, and the Hilbert transform was performed to extract the characteristic signals.

After demodulation, the input features constructed from the input features is given as

$$T_1^1 = [SES_{s1}^1, SES_{s2}^1, \dots, SES_{s8}^1]$$

$$T_2^1 = [SES_{s2}^1, SES_{s3}^1, \dots, SES_{s8}^1]$$

$$T_3^1 = [SES_{s3}^1, SES_{s4}^1, \dots, SES_{s8}^1]$$

$$\vdots$$

$$T_8^1 = [SES_{s8}^1, SES_{s1}^1, \dots, SES_{s7}^1]$$

Fig. 12. The identification of gear fault with different numbers of GMMs and states in TR#5. Then, we used these input features to train the HMM

model. The HMM used here is a 5-state model with a diagonal covariance matrix containing a mixture of 3 Gaussian models. Once the HMM of the normal condition is trained, the trained HMM will be used for the whole life of the test with other  $T^j$ s being the validation data. Finally, we construct API by (17) to reflect the health condition of the gearbox.

#### 4.3.2 Evaluation of API using TR#5 under constant number of states and GMMs.

In this section, we will evaluate API and compare it with other fault growth parameters, including FGP1 and KI. FGP1 is defined as the part (percentage of points) of the residual error signal that exceeds three standard deviations calculated from the baseline residual error signal [14]:

$$FGP1 = 100 \sum_{i=1}^L \frac{w_i}{W} I(r_i > \bar{r} + 3\sigma_0), \quad (19)$$

where

$$w_i = I(r_i \leq \bar{r} + 3\sigma_0) + \left( \left\lfloor \frac{r_i - \bar{r}}{3\sigma_0} - 1 \right\rfloor + 1 \right) I(r_i > \bar{r} + 3\sigma_0)$$

$$W = \sum_{i=1}^L w_i, \quad (20)$$

Here,  $r_i$ 's are the current residual error signal points,  $\bar{r}$  is the mean value of the current residual signal,  $\sigma_0$  is the “baseline” standard deviation,  $I(\cdot)$  is the indicator function, and  $\lfloor \cdot \rfloor$  is the floor function. In addition, another index based on the kurtosis of the residual error signal is presented here for comparison, which is called the Kurtosis Index (KI). The kurtosis usually increases if the signals are properly filtered, and the kurtosis of the residual error signal has been proven to give the earliest indication of gear tooth fault [15].

Then, we used vibration data from TR#5 to evaluate the performance of FGP1, KI, and API for gearbox health evaluation. In TR#5, two adjacent teeth on the drive gear were broken after test-rig shutdown. The total number of running hours was 127.4 hours, including 83 data files. In order to keep a constant workload, we chose data with file numbers from 13 to 83, which corresponded to all vibration data collected under Condition #2.

The health evaluation results for the gear fault are shown in Fig. 9. It should be emphasized that the decrease in API represents the deterioration of the machine health condition, while for FGP1 and KI, the increase of the health index represents deterioration. First, we can find that all three indexes can describe the gear health condition. However, the comparison results show that the health trend described by API gradually deteriorates and KI remains almost constant, while the fluctuation of FGP1 is obvious before the occurrence of an early fault. This means that the visual inspection of API and KI is better than FGP1. Since API has its own threshold (red cross marker line), it can identify the occurrence of early faults automati-

cally. Therefore, we can conclude that API with its semi-dynamic threshold is the best of the three indexes under constant output torque.

#### 4.3.3 Evaluation of API using TR#10 under constant number of states and GMMs

TR#10 was shut down with distributed gear teeth broken on the drive gear at the end of the test. The total number of running hours was 189.25 hours, including 148 data files. Similarly, we chose data files with numbers from 13 to 148. The comparison results are shown in Fig. 10. In Fig. 10, we also see that all three indexes can reflect gear health condition. For FGP1 in Fig. 10(a) it is hard to precisely identify the occurrence of a fault. Moreover, in Fig. 10(b), KI exhibits irregular fluctuation as there is a great change at file number 40 that causes severe distortion for visual inspection. On the other hand, Fig. 10(c) shows that API with its semi-dynamic threshold can detect the occurrence of an early fault in the gear at file number 40. This led us to conclude that API with its semi-dynamic threshold was the best among the three condition monitoring indexes.

#### 4.3.4 Evaluation of API with different number of states and GMMs

It is known that the number of states and GMMs can have an impact on the accuracy of fault diagnosis with HMM. However, in most studies on Hidden Markov Models for fault diagnosis, no detailed discussion is given on how to choose the optimal number of states and GMMs. In order to investigate this problem, an experiment was conducted to vary the number of GMMs under different states based on the work in Section 4.3.2. At the same time, we found that the maximum number of GMMs decreased when the number of states increased. Fig. 11 plots the relationship between the maximum number of GMMs and the number of states in this study.

In the following experiments, we fix the number of states at 2 and vary the number of GMMs from 2 to the maximum. Similar to Fig. 9(c), we found that API detected the occurrence of early faults in TR#5 at file number 72 with different GMMs. Further, we varied the number of states from 2 to 7 and repeated the procedure above. Fig. 12 shows the occurrence of an early fault in TR#5 at file number 72.

The same experiment was performed in TR#10. The results indicated the occurrence of early faults at file number 40. From Fig. 12 and Fig. 13, we conclude that API has less sensitivity to the selection of number of GMMs and states. In other words, this proposed index can avoid the problem concerning the selection of the optimal number of GMMs and states in the application of HMM with a mixture of Gaussian outputs.

## 5. Conclusions

In this paper, we developed an adaptive health evaluation index called Average Probability Index (API) based on EMD and HMM. Compared with other research in fault diagnosis

with EMD and/or HMM, our purpose was to realize online health evaluation. Therefore, details were given to show the construction of a health index and its threshold criteria. Two sets of vibration data collected from MDTB were used to validate the proposed index with its dynamic threshold. The analysis results showed that API can well describe the health condition of gears. The advantages of this index can be summarized as follows:

The proposed index can be regarded as an adaptive one since EMD is an adaptive procedure, compared with Wavelet Transform, Wavelet Packet Transform, and Wavelet Lifting Scheme.

API gives a quantitative description of machine health condition, which can be used for estimation of machinery remaining life and maintenance decision analysis.

API has less sensitivity to the selection of the number of GMMs and states.

The proposed method needs very few samples for model training. Therefore, the dimensionality of data is small, and it is appropriate for online condition monitoring with less computation time.

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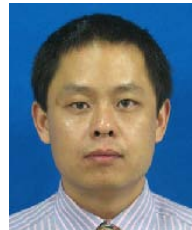
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